

## CONCEPTUAL CONNECTIONS BETWEEN STUDENT NOTICING AND PRODUCTIVE CHANGES IN PRIOR KNOWLEDGE

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*In this report, I examine what students noticed as they participated in an instructional unit on quadratic functions and how a shift in what they noticed was conceptually connected to productive changes in their prior knowledge about linear functions. My results show that, over the course of the instructional unit, students' attention shifted toward noticing changes in the quantities involved in quadratic functions. Furthermore, I identify two conceptual connections between the shift toward noticing changes in quantities in quadratic contexts and the productive changes in reasoning that students exhibited on post-interview linear function tasks.*

**Keywords:** Algebra and Algebraic Thinking; Design Experiments; Mathematical Knowledge for Teaching

### Purpose of the Study

Multiple studies in mathematics (and science) education have examined aspects of how prior knowledge changes as new knowledge is acquired (e.g., Hohensee, 2012; Smith, diSessa, & Roschelle, 1993; Vosniadou & Brewer, 1987). My study extends the research in this area by identifying explanations for why prior knowledge changes in productive ways as a consequence of new knowledge being acquired about a related but different topic. The purpose of the current research is to account for the role that features of the instructional environment played in bringing about the previously reported finding that five out of seven middle school students that participated in an instructional unit on quadratic functions exhibited productive changes in their prior understanding of linearity as a result of participating in the unit (Hohensee, 2012).

In the earlier research in which productive changes in prior knowledge were observed, students went from reasoning in non-proportional ways with the quantities from linear functions prior to participating in a quadratic functions instructional unit, to reasoning proportionally with the changes in linear function quantities after the unit. For example, in the pre-interview, when students were shown the graph of the linear relationship between the hours a cell phone was used and the cost of using the cell phone, Jenn,<sup>1</sup> one of the participants, reasoned *univariately* (i.e., she reasoned exclusively with the cost variable) and concluded incorrectly that the rate of the function was non-constant. However, on a post-interview task about the graph of a linear relationship between the number of employees in a business and the cost of running the business, Jenn reasoned proportionally with the changes in quantities (i.e., after finding a 4 employee change in the number of employees from 8 to 12 employees and a corresponding \$2500 change in cost from \$6000 to \$8500, she multiplied the rate of \$625 per employee by 4 employees to see if it produced the \$2500 change in cost).

I considered various explanations for why students' prior knowledge about linear functions had changed as a result of participating in instruction on quadratic functions. Of the explanations I considered, *student noticing* offered the greatest promise as an underlying mechanism behind the productive changes. According to Lobato, Rhodehamel and Hohensee (2012), student noticing is defined as "selecting, interpreting, and working with particular mathematical features or regularities when multiple sources of information compete for students' attention" (p. 9). One important reason why student noticing recommended itself to my purposes was because research has already shown that student noticing possesses explanatory power for changes in how students think about novel tasks as a result of instruction (Lobato et al., 2012). Although what students notice and what students understand in a particular context is likely closely related, looking at what students notice offers a unique perspective on their thinking. In this paper, I examine what students noticed during an instructional unit on quadratics to see if it is conceptually connected to the productive changes in their prior knowledge about linear functions.

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### Theoretical Foundation: Noticing and the Focusing Framework

Much cognitive and psychological research has examined attention (e.g., McCandliss, Beck, Sandak, & Perfetti, 2003; Posner & Fan, 2008; Treisman & Gelade, 1980). However, only a small body of mathematics education research has examined attention from the perspective of what students notice in more realistic educational settings (e.g., Lobato et al., 2012; Radford, Bardini, & Sabena, 2007). Building on the limited prior work on student mathematical noticing, I used Lobato et al.'s definition of *noticing* as stated above. Furthermore, I used Lobato et al.'s four-part *focusing framework*, which was specifically designed to characterize what students notice in mathematics instructional settings. The four parts are: (a) the centers of focus, which are the objects that students attend to within a given perceptual or conceptual domain; (b) the focusing interactions, which are the discursive practice that influence what students notice; (c) the features of the mathematical tasks, which are the attributes of the activities that students participate in that “afford and constrain” (Lobato et al., 2012, p. 12) what they notice; and (d) the nature of the mathematical activity, which is the participatory structure of the classroom environment (i.e., the norms that get established in the classroom). The center of focus represents the psychological aspect of noticing while the other three parts refer to the social structures of mathematics classrooms that influence what gets noticed. Thus, the focusing framework coordinates the psychological and the social, to develop a comprehensive picture of student noticing in realistic educational settings.

### Methods

I employed a design-based research (DBR) methodology for this study (Design-Based Research Collective, 2003). Specifically, my quadratics instructional unit became the third iteration of the unit, the previous two iterations being part of a larger study conducted by the research team with which I was associated.<sup>2</sup> Thus, I continued the refinement of the activities that had been developed in the previous two iterations. My instructional unit was also similar in duration to the previous iterations (16 hours of instruction, spread over two weeks).

Seven students were recruited from an ethnically diverse urban middle school set in a middle class neighborhood. I, the author of this paper, served as the teacher. This is consistent with the principles of DBR in education, where the line between the teacher and researcher is often blurred to make in-the-moment refinements of the instructional design possible (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003).

Each class was recorded by two video cameras operated by graduate students and another researcher. The camera operators also served as external observers, presenting their observations to the teacher during debriefing sessions at the end of class. These observations further contributed to the refinement of the instructional unit. Incorporating observer feedback while the instructional unit is still being conducted is consistent with the principles of DBR (Steffe & Thompson, 2000).

I began data analysis by creating a descriptive account of the things that were said by students and the teacher with minimal interpretation (Miles & Huberman, 1994). I also identified episodes that seemed potentially rich with respect to student noticing during this pass through the data. To prevent data overload, the data was then reduced (Miles & Huberman, 1994) to these rich episodes. Next, multiple analytic passes were made through the reduced data, one pass for each part of the focusing framework. In the first pass, I identified the emergent *centers of focus* (i.e., what students appeared to be attending to mathematically). In the second pass, I used a priori codes to categorize the *focusing interactions* that occurred in and around the time in which each center of focus emerged. In the third analytic pass, I identified a connection between the features of the *mathematical tasks* that students engaged in and the centers of focus that emerged. In the fourth pass, I analyzed the *nature of the mathematical activity* that appeared to be related to the centers of focus that emerged in the instructional intervention.

Finally, I looked for conceptual connections between what students were noticing (their centers of focus) and productive changes that I had discovered in the students' prior knowledge about linearity during an earlier analysis of the post-interview (Hohensee, 2012). Looking for conceptual connections is consistent with the *realist view of causation* (Maxwell, 2004). Researchers who subscribe to a *realist view*

assume that the actual causal mechanisms (processes) underlying regularities between events can be observed. In contrast, researchers who subscribe to the *regularity view of causation* assume that the causal mechanisms that underlie events are unobservable. The realist view aligns with taking a *process-oriented approach* to research (i.e., conducting qualitative research and creating causal explanations). Therefore, in looking for conceptual connections between noticing and productive changes in prior knowledge, I oriented my analysis toward noticing as a potential process underlying the productive changes in prior knowledge I had previously discovered.

## Results

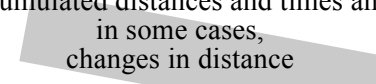
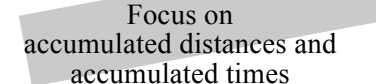
In this section, I present classroom evidence that shows there was a shift over the course of the quadratics instructional unit in what students noticed (see Table 1). Three types of evidence from the classroom data will be presented: (a) evidence of students' initial center of focus prior to the shift; (b) evidence of the new center of focus after the shift; and (c) evidence of the how the shift in center of focus was socially organized by particular kinds of focusing interactions, features of the mathematical tasks and nature of the mathematical activity (see Table 2).

### Initial Centers of Focus

Students initially appeared to be focused on accumulated distances and accumulated times and in some cases on the changes in distance as well. Kendra,<sup>3</sup> Jenn, Armando, and Nicolas were initially focused on all three quantities. For example, on a Lesson 2 task, which involved a SimCalc Mathworlds™ computer simulation of a fish swimming according to a quadratic distance-time function, Kendra wrote, "From 0-2, it's 1 second. And then in the 2nd second it goes 6 feet, in the 3rd second it goes 18 feet, which is 10 feet. Then in the 4th second it goes 32 feet, which is 14 feet. In the 5th second it goes from 32 feet to 50, which is 18 feet." In this response, Kendra focused on the accumulated distances (i.e., 2, 18, 32 and 50 ft), the changes in distance (2, 6, 10, 14 and 18 ft) and the accumulated times (i.e., 1st, 2nd, 3rd, 4th and 5th second). To an adult, Kendra's talk about the 1st or 2nd second may seem like she was also attending to the 1-second changes in time. However, previous work in this area has shown that, unless middle school students explicitly refer to the 1-second changes in time (e.g., the fish went 6 ft in 1 second), they are likely not focused on those quantities (Lobato et al., 2012).

Other students like Peter, George, and Brady appeared to be initially focused on accumulated distances and accumulated times only. For example, on the same task described above, George and Brady recorded accumulated time/accumulated distance fractions (i.e., 1 s/2 ft, 2 s/8 ft, 3 s/18 ft, 4 s/32 ft, 5 s/50 ft, 6 s/72 ft, and 7 s/98 ft) and then reduced all to equivalent fractions with a 1 in the numerator (i.e., 1/2, 1/4, 1/6, 1/8, 1/10, 1/12 and 1/14). Despite George and Brady reducing their time/distance fractions to a numerator of 1, I did not count this as an instance of focusing on 1-second changes in time and the corresponding changes in distance because (a) they dropped the units for the reduced fractions; (b) they did not refer to the numerators as representing 1-second changes in time or to the denominators as changes in distance; and (c) they produced fractions that represented the set of *average distances* the fish travelled in the first 1, 2, 3, 4, 5, 6 and 7 seconds, rather than the changes in distance over each second.

**Table 1: Summary of Shifts in Centers of Focus**

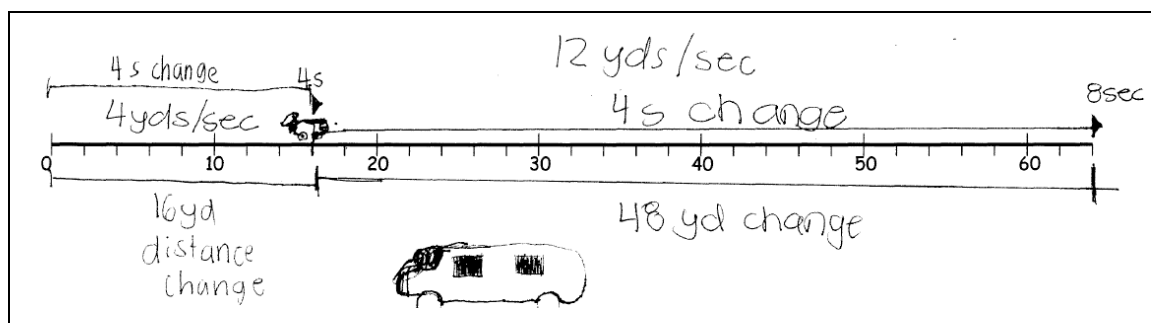
Lesson when shift occurred	Student	Initial Center of Focus	New Center of Focus
Lesson 2	Jenn	 Focus on accumulated distances and times and, in some cases, changes in distance	Focus on changes in distance and changes in time
Lesson 8	Armando		
Lesson 9	Nicholas		
Lesson 11	Kendra		
		 Focus on accumulated distances and accumulated times	
Lesson 8	Peter		
Lesson 8	George		
Lesson 8	Brady		

**Table 2: Summary of Social Organization of Individual Noticing**

Individual component of shift in noticing	Social component of shift in noticing		
Center of focus after shift	Focusing interactions	Features of the mathematical tasks	Nature of the mathematical activity
<ul style="list-style-type: none"> <li>Focusing on changes in distance <i>and</i> changes in time</li> </ul>	<ul style="list-style-type: none"> <li>Naming</li> <li>Highlighting</li> </ul>	<ul style="list-style-type: none"> <li>Specifically targeting an exploration of changes in quantities</li> <li>Varying the relationships between changing quantities</li> </ul>	<ul style="list-style-type: none"> <li>Presenting diagrams</li> <li>Noticing features in others' diagrams</li> <li>Asking <i>Can you explain why?</i> questions</li> </ul>

### New Center of Focus

As Table 1 shows, all students exhibited a shift in their center of focus with respect to the quantities they noticed in quadratic function distance-time data. Specifically, by Lesson 8, six students had converged on noticing the changes in distance and the changes in time. The other two students began consistently recording changes in distance and time in their diagrams for quadratic distance-time functions by lessons 9 and 10, respectively. One way that students exhibited this new center of focus was by recording changes in distance and time in their diagrams of quadratic distance-time data. For example, in Lesson 8, when students were presented with tabular quadratic distance-time data representing the motion of a remote-control (RC) car (i.e., [0 s, 0 yds], [4 s, 16 yds], [8 s, 64 yds]) and asked to produce a diagram showing speeds, George, Armando, Jenn, Brady, and Peter recorded changes in distance and the corresponding changes in time in their diagrams (see Figure 1 for Peter's diagram).



**Figure 1: Peter's RC car diagram highlighting changes in distance and time**

Students also verbally referred to changes in distance and time when explaining or reasoning about their own and other's diagrams. For example, when Kendra explained what she noticed in Brady's diagram, which had been projected for the class to see, she said, "Umm, he put the *change in time* like I think it's like the box thing and then the *change in distance* on the bottom where 0 to 16 yards. And then over here [points to another part of his diagram] he did the same." Jenn used similar language when describing her diagram:

I have the *change in time* up here [points at change in time labels] 1 second for all of them, and *change in distance* [points at change in distance labels] and you can see like it's getting faster because the blocks are bigger [points from left to right].

Except for Armando, each student provided at least one example of similar dialogue.

After Lesson 8, there were only two instances in which students appeared to *not* focus on changes in distance and time. One instance occurred during Lesson 10, when Armando recorded changes in distance on his diagram, but not changes in time. The other instance occurred during Lesson 16, when Peter did not record changes in distance or time on his diagram. However, his diagram was not completed. It is possible that he would have added them had he had more time.

The new center of focus represents the psychological aspect of student noticing that emerged during the quadratics instruction. Next, I provide evidence of the interactions and educational structures that represent the social aspect of what students noticed. Later, I discuss how the new center of focus was conceptually connected to the productive changes in students' prior knowledge for which noticing is posited as an underlying process.

### Focusing Interactions

Several kinds of focusing interactions appeared to influence what students noticed. In particular, analysis of the data revealed that *naming* and *highlighting* contributed to the shift in the center of focus.

**Naming.** The teacher used naming, which is defined as "the act of using a category of meaning from mathematical practice to classify and label some mathematical characteristic or property" (Lobato et al., 2012, p. 35), in introducing the new quantities *changes in distance* and *changes in time* (Lesson 3). In particular, the teacher said:

So now I wanna ask you something . . . So if this is how clown or rabbit runs, could anybody see a place where the change in distance is 6 when the change in time is 1? . . . So maybe think about that just a second. I'll say it again. Can anybody see a place where the change in distance is 6 when the change in time is 1?

After introducing these new names and talking about them, the teacher encouraged students to use the new names:



OK, so see if you can use the words that I'm using. I said what's the change in distance, or sorry, can you see a change in distance of 6 when there's a change in time of 1? So see if you can use the word "change" in how or what you're saying.

This kind of encouragement appeared to be effective because all students began using both names regularly in small- and whole-group discussions.

A hypothesis for why the names *changes in distance* and *changes in time* may have influenced what students noticed is that the names appeared to evoke for students a strong image of making comparisons. For example, the first time the teacher asked, "Can anybody see a place where the *change in distance* is 6 when the *change in time* is 1?" all but one student verbalized or gestured in a way that indicated they were making comparisons between two distances and/or two times (e.g., Peter and Jenn pointed from one distance to another on a number line; Armando said "The distance changes from 6 to 18 by 12 meters and 2 seconds"). This hypothesis is consistent with other findings that have shown that mathematical terms can create powerful images (e.g., Siebert & Gaskin, 2006).

**Highlighting.** The teacher and students used *highlighting*, which is defined as "methods used to divide a domain of scrutiny into a figure and a ground, so that events relevant to the activity of the moment stand out" (Goodwin, 1994, p. 610), to foreground changes in distance and changes in time on student-generated diagrams. An example of gestural highlighting occurred when Jenn, who was explaining to the other students her number-line diagram representing the distance and time for a swimming fish, swept her finger back and forth between positions of the fish to highlight particular changes in distance and time as she said, "I have the *change in distance* and the *change in time*, from each point [points to change in distance and change in time labels] that takes *one second*." An example of written highlighting occurred when the teacher annotated student work with arrows that highlighted the changes in distance and time and then projected the student work for the class to see and discuss.

In each of these examples, highlighting foregrounded the changes in distance and time and likely backgrounded accumulated quantities. In summary, highlighting and naming appeared to be focusing interactions that figured prominently in the emergence of the new center of focus on the changes in distance and changes in time.

### Mathematical Tasks

There were at least two features of the mathematical tasks that likely contributed significantly to the emergence of the center of focus on changes in distance and changes in time. A common feature of tasks in Lesson 3 and Lesson 8 was that students were explicitly asked to find changes in distance and time. A common feature of tasks in Lessons 8, 9 and 10, was that they all involved students drawing diagrams of the same quadratic distance-time data, but with different equal partitions of the time variable (i.e., over 4 s, over 2 s and over 1 s intervals). Whereas the first feature explicitly directed attention to changes in distance and time, this second feature likely directed attention to changes in time. Interestingly, it was during Lessons 8, 9 and 10 that all students came to focus on changes in quantities. Thus, changes in distance and time appeared to become particularly salient for students during these tasks.

### Nature of the Mathematical Activity

I characterized the nature of the mathematical activity that likely contributed to the shift in what students noticed as defining three related classroom norms (Cobb & Yackel, 1996): (a) presenting student work to the class, (b) noticing features of other students' diagrams, and (c) students asking each other *can you explain why?* The norm of presenting work provided students with examples of what others were noticing when creating their diagrams. In particular, students provided verbal evidence that they saw how others were recording changes in distance and time. Furthermore, the norm of noticing features of each other's diagrams meant that students publically identified the recorded changes in distance and time that they noticed on each other's diagrams. Finally, the norm of asking each other *explaining why* questions meant that students engaged in a close scrutinization of the quantities that were represented in each other's diagrams.

## Discussion

In the results section, I provided evidence that all seven students in my study shifted their center of focus toward noticing changes in distance and time over the course of the instructional unit. Furthermore, I showed that particular focusing interactions, features of the mathematical tasks and the nature of the mathematical activity appeared to influence their shifts. However, an additional goal of my study was to determine if what students noticed in the instructional unit was linked to the productive changes I observed in five of the seven students' prior knowledge about linearity when I compared their understanding before and after the quadratics instructional unit. To achieve this goal, I compared what students noticed during instruction with the productive changes that five students exhibited when they reasoned proportionally with changes in quantities on linear function tasks during their post-interviews. This comparison led me to discover two important conceptual connections.

The first conceptual connection was that noticing changes in quantities in a quadratic context and reasoning proportionally with changes in quantities in linear function contexts both involve the same focus on changes in quantities. In other words, the new center of focus that emerged during instruction on quadratics persisted into the post-interviews, where students reasoned proportionally with changes in quantities in linear contexts. For example, on a post-interview linear function task about a water-pump, all but one student recorded both the changes in water volume and the corresponding changes in time. Thus, changes in quantities, which were established as a new center of focus in the instructional intervention, also appeared to be a center of focus during the post-interview.

Second, the students who appeared to most quickly establish a focus on both changes in distance and changes in time during quadratics instruction provided the greatest increase in proportional reasoning with changes in quantities on linear tasks in the post-interview. In the instructional intervention, Jenn and Nicholas were the first students to attend explicitly to the changes in distance and changes in time; George, Armando, Brady, and Peter required an extra lesson before they also began to focus on changes in distance and time; Kendra required three extra lessons. In the post-interview, Jenn, Nicholas, and Brady changed from reasoning non-proportionally to reasoning proportionally with changes in quantities, while Peter, George, and Armando's reasoning changed less, and Kendra provided no evidence of reasoning with changes in quantities. Therefore, the quicker the establishment of a focus on changes in distance and time, the greater the productive change in prior knowledge seemed to be. Brady was the exception because he exhibited similarly substantial changes in reasoning during the post-interview as Jenn and Nicholas did, despite not attending as quickly to changes in distance and time in the instructional intervention.

These conceptual connections suggest that noticing is an underlying mechanism for productive changes to prior knowledge that occur as a result of learning something new. Nevertheless, further investigation into the connection between student noticing and productive changes in prior knowledge, which has thus far been under-researched, is warranted.

## Endnotes

<sup>1</sup> All participant names are gender and ethnicity preserving pseudonyms.

<sup>2</sup> NSF-funded 3-year collaboration between researchers at San Diego State University and University of Wisconsin-Madison (Joanne Lobato, PI; Grant REC-0529502).

<sup>3</sup> Participant names are gender and ethnicity preserving pseudonyms.

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